Int. J. Multiphase Flow, Vol. 3, pp. 585–596. Pergamon/Elsevier, 1977. Printed in Great Britain

A MODEL FOR SLUG FREQUENCY DURING GAS–LIQUID FLOW IN HORIZONTAL AND NEAR HORIZONTAL PIPES

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(*Received* 25 *August* 1976; *received for publication* 24 *May* 1977)

Abstract—Slug formation is an entry region phenomena. Waves form on a growing stratified film eventually blocking the gas to form a slug. The liquid level drops when the slug is swept away and the waves disappear. Then the film rebuilds its level in a time equal to the inverse frequency. The process is modelled and the results shown to be in agreement with experiment.

INTRODUCTION

When gas and liquid flow simultaneously in a pipe, various flow patterns may result. One of the most common of these is slug flow in which coherent plugs of aerated liquid are separated by regions of gas riding on a thin liquid film. Pressure drop during slugging may be an order of magnitude higher than would be the case if the flow were homogeneous or stratified.

It is quite important first to be able to predict the conditions at which slug flow will take place and second to calculate pressure drop for this complicated flow pattern. The former problem has been analyzed by Taitel & Dukler (1976). The latter has been successfully approached by Hubbard & Dukler (1965) who modelled steady slug flow and obtained accurate predictions for the hydrodynamic behavior including slug and film lengths, velocities and pressure drop. However, their model is not complete since it requires the slug frequency as input data. The theory shows the pressure drop is almost directly proportional to the slug frequency. Thus, it is important to be able to predict this quantity.

Determining this frequency requires the analysis and understanding of the mechanism of slug formation. Visual observations reveal that the slug is created as a result of unsteady waves formed on a growing stratified film which blocks the air passage. The entire phenomenon occurs near the entrance region and as such it can be classified as an entrance phenomenon in contrast to steady slug flow which has been shown to be independent of entry configuration. These observations further indicate that whenever slug flow exists the streams are stratified at the inlet and the slug formation phenomena can be described as follows:

> Stratified liquid and gas flow into the pipe. The picture just after a slug passes to the right is shown in figure 1A. The liquid decelerates and its level rises gradually until it reaches an equilibrium level. This level is determined by the forces at the interface and those at the walls bounding the two phases as discussed below. The equilibrium surface is unstable and solitary waves form and grow as in figure 1B. Eventually one wave blocks the air passage as shown in figure 1C. As soon as the bridge is formed the liquid is accelerated by the gas, sweeping the liquid in front of it forming a slug as in figure 1D. As a result, just downstream of the point of closure the level is lower than the equilibrium level. The hydrostatic forces due to this difference cause flow into the film and it rebuilds to its original level as shown in 1A, thus starting a new cycle.

Due to the multitude of factors influencing the slug frequency, researchers in the past have been careful to suggest that use of their results be limited to the range of experimental conditions for which the data were taken. Most of the authors report their data in the simple form of frequency as a function of liquid and gas flow rates for fixed pipe diameter and fluid

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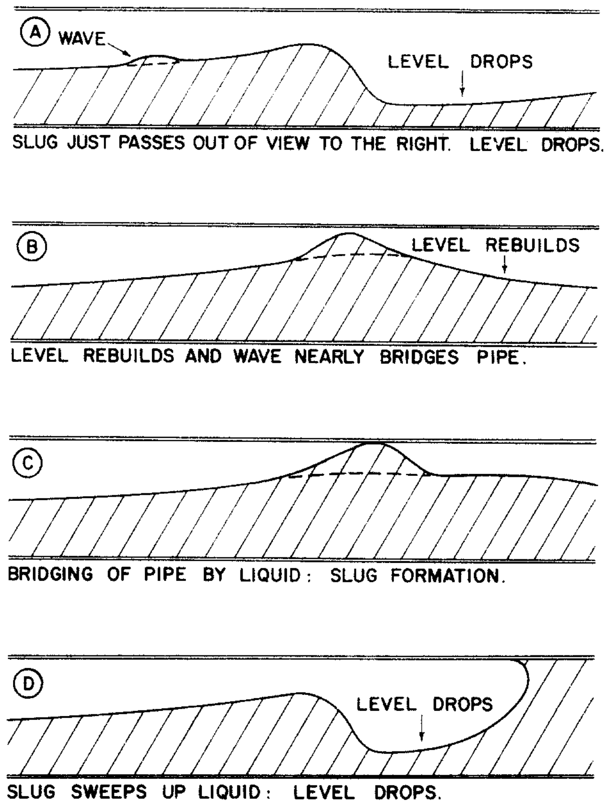

Figure 1. The process of slug formation.

properties (Dukler & Hubbard 1975; Kordyban & Ranov 1963; Chu 1968; Vermeulen & Ryan 1971). Recently, however, there has been an attempt to arrive at more general correlations (Gregory & Scott 1969; Grescovich & Shrier 1972) without success. These correlations are not reliable for use at flow conditions different than the ones used in their construction.

In this work the complicated entrance phenomenon is modelled using open channel flow equations for the purpose of analytically predicting the slug frequency as well as to improve the understanding of slug initiation phenomenon.

ANALYSIS

The objective of the analysis is to calculate the time, θ, between two successive wave closures which form slugs. The frequency is then $1/\theta$. Although measurements show that there is some randomness in this cycle time the variance has been shown to be small (Dascher 1970) and for the purposes of this work a fully deterministic model is used with a mean value of θ between all slugs.

Starting from the instant at which a wave bridges the pipe the process can be visualized in two steps, each requiring a time for completion.

Time to sweep away the liquid in front of the wave and for the level to drop to its lowest value, θ_1

Time to build to the equilibrium level, θ_2

Observation shows that θ_1 is very short compared to the cycle time. Furthermore, it is observed that almost immediately after the level rebuilds to its equilibrium value a solitary wave formed upstream moves across the surface and closes the air gap. The frequency of such solitary waves is an order of magnitude larger than the slug frequency so that effective closure takes place immediately after the equilibrium level is reestablished. In the event that premature closure takes place (i.e. a wave causes closure before the level has rebuilt to h_e) a nonpersisting slug is formed that either disappears or merges with a previous slug. Thus it is suggested that the rebuild time θ_2 is the characteristic time, θ, for this cyclic phenomena. The problem of predicting slug period or frequency reduces to determining the time, θ_2, to rebuild the film from its lowest level, h_s, just after a slug has been formed, to the equilibrium level, h_e. Predicting θ_2 requires the application of the conservation equations for mass and momentum as in an open channel flow and the determination of the levels h_s and h_e.

Equations for rebuilding the liquid film

Figure 2 illustrates the process of rebuilding the film. At $t = 0$, $x = 0$, the level is pictured as having a step change from $h = h_e$ to $h = h_s$. For $t > 0$ rebuilding of the film takes place as shown. Eventually at some point in time and space the level will exceed h_e and this is considered the time, θ_2.

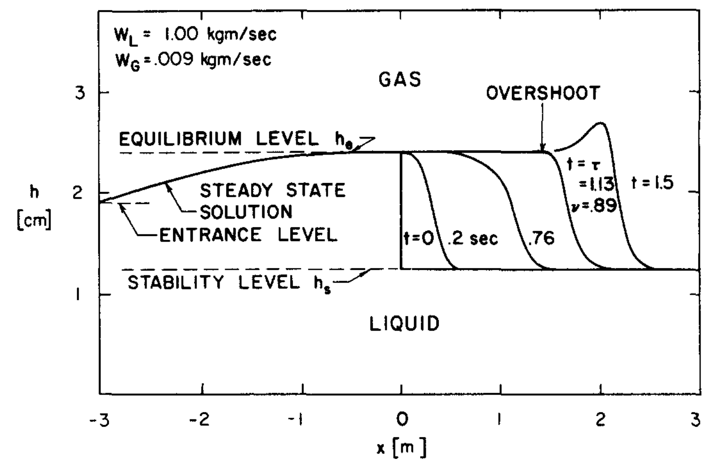

Figure 2. Solution for film buildup. Water–air in a 3.81 cm dia. pipe.

The integral momentum and continuity equations for the liquid phase averaged over the flow area are given by

$$\rho_L \frac{\partial(u_L A_L)}{\partial t} + \rho_L \frac{\partial(u_L^2 A_L)}{\partial x} = -\tau_L S_L + \tau_i S_i - A_L \rho_L g \cos\alpha \frac{\partial h}{\partial x} - A_L \frac{\partial P}{\partial x} + \rho_L A_L g \sin\alpha, \qquad [1]$$

$$\frac{\partial A_L}{\partial t} + \frac{\partial(u_L A_L)}{\partial x} = 0 \qquad [2]$$

where u_L is the average liquid velocity, A_L the cross sectional area of the liquid, P pressure, ρ_L liquid density and α is the angle between the pipe axis and the horizontal (positive for downward flow) and g is the gravitational acceleration. The cross sectional area of the liquid film depends on the level of the liquid in the pipe. Thus, using $A_L = A_L(h)$ [1] and [2] take the following form

$$g \cos\alpha \frac{\partial h}{\partial x} + \frac{\partial u_L}{\partial t} + u_L \frac{\partial u_L}{\partial x} = -\frac{\tau_L S_L}{\rho_L A_L} + \frac{\tau_i S_i}{\rho_L A_L} - \frac{1}{\rho_L}\frac{\partial P}{\partial x} + g \sin\alpha, \qquad [3]$$

$$\frac{\partial h}{\partial t} + u_L \frac{\partial h}{\partial x} + \frac{A_L}{A_L'}\frac{\partial u_L}{\partial x} = 0, \qquad [4]$$

where A_L' represent differentiation with respect to h.

For a rectangular pipe $A_L = hb$, where b is the width of a rectangular pipe; for circular pipes the cross sectional area as a function of the elevation from the bottom of the pipe, h, is given by

$$A_L = r^2\left[\pi - \cos^{-1}\left(\frac{h}{r}-1\right) + \left(\frac{h}{r}-1\right)\sqrt{\left(1-\left(\frac{h}{r}-1\right)^2\right)}\right] \quad [5]$$

where r is the pipe radius. τ_L represents the frictional shear stress between the liquid and the solid walls of the pipe and τ_i the forward stress at the interface caused by the faster moving gas over the liquid surface. S_L and S_i are the contact perimeters between the liquid and wall and of the liquid–gas interface respectively and are given by

$$S_L = 2r\left[\pi - \cos^{-1}\left(\frac{h}{r}-1\right)\right], \quad [6]$$

$$S_i = 2r\sqrt{\left[1-\left(\frac{h}{r}-1\right)^2\right]}. \quad [7]$$

To calculate τ_L and τ_i a conventional method is followed with the additional simplification that the gas velocity is much larger than the liquid velocity and that the liquid interface is smooth. In this case

$$\tau_L = f_L\frac{\rho_L u_L^2}{2} \qquad \tau_i = f_G\frac{\rho_G u_G^2}{2}. \quad [8]$$

For the friction factors f_L and f_G we use a Blasius type correlation for both the liquid and the gas

$$f = CR_e^{-n} \quad [9]$$

where C and n are chosen according to the condition of flow (turbulent or laminar) that exist in the liquid or gas. In this work the case was considered of both fluids being turbulent for which $C = C_L = C_G = 0.046$ and $n = n_L = n_G = 0.2$. The Reynold number is based on the hydraulic diameter. For this purpose the liquid is treated as if it flows in an open channel and $D_L = 4A_L/S_L$ whereas the gas is visualized as flowing in a closed duct and thus $D_G = 4A_G(S_i + S_G)$.

Next consider the pressure term $\partial P/\partial x$ in [3] and turn attention to the equation of motion for the gas film. Note that the pressure drop in the liquid and the gas is assumed to be equal at the interface. Equations [1] and [2] are, in principal, equally valid for the flow of the gas film. However, in the gas region we neglect gravity effects related to the change of the cross sectional area. Furthermore, since the gas velocity is much larger than the velocity of the liquid, a quasi steady state for that phase can be assumed. In this case we obtain for the gas region

$$\rho_G\frac{\partial(u_G^2 A_G)}{\partial x} = -\tau_G S_G - \tau_i S_i - A_G\frac{\partial P}{\partial x} + \rho_G A_G g \sin\alpha \quad [10]$$

$$A_G u_G \rho_G = W_G = \text{const.} \quad [11]$$

where W is mass flow rate and the subscript G indicates gas.

Equations [10–11] result in

$$\frac{\partial P}{\partial x} = -\tau_G\frac{S_G}{A_G} - \tau_i\frac{S_i}{A_G} - \rho_G\left(\frac{W_G}{\rho_G A_G}\right)^2\frac{A_L'}{A_G}\frac{\partial h}{\partial x} + \rho_G g \sin\alpha. \quad [12]$$

The first 2 terms on the R.H.S. of [12] are related to the frictional pressure drop in the gas. The fourth term represents the axial force per unit volume due to gravity. The third term can be designated as the "Bernoulli Effect." Upon substituting [12] into [3], it can be shown that this term causes the gas–liquid interface to be unstable to small disturbances once

$$g \leq \frac{\rho_G}{\rho_L}\left(\frac{W_G}{\rho_G A_G}\right)^2 \frac{A_L'}{A_G}. \qquad [13]$$

The "Bernoulli" term describes the creation of unstable waves as a result of decreased pressure above the wave due to gas acceleration. Retention of this term causes insurmountable mathematical difficulties since it can be demonstrated that no unique unsteady state solution exists when [13] is satisfied. However, consistent with our approach of studying the rebuilding and drainage cycle of the mean level of the liquid film and disregarding the wave formation in the analysis, we can neglect the "Bernoulli term" in this part of the solution. As further justification for neglecting this term, it should be noted the solution of [3] and [4] is developed only when $\partial h/\partial x$ is negative. Under this condition, the gas passage is a divergent channel and when that is the case the presence or growth of waves cannot significantly influence the process of film growth. By recognizing the unimportance of the wave growth problem to the modelling of the process of rebuilding the film, it is possible to avoid an intractable mathematical difficulty.

Equations [3] and [4] are two simultaneous partial differential equations for $h(x, t)$ and $u_L(x, t)$. Note that the R.H.S. of [3] is a known function of h and u_L. Solving to find the time required for the level to move from h_s to h_s requires only the definition of these two states.

The equilibrium level, h_e

The equilibrium level h_e can be determined independently using a simple (though iterative) solution of an algebraic equation which results by equating the R.H.S. of [3] to zero. This procedure is equivalent to equating the pressure drop of the gas and liquid in stratified equilibrium flows (Taitel & Dukler 1976). Since the slug is formed at a point where steady state has already been reached, this suggests that slug frequency is independent of entrance geometry. This indeed has been observed experimentally (Hubbard 1965).

The level, h_s

An intuitive approach to predicting h_s is to assume that it is controlled by the hydrodynamics of the slug which was just formed. It would then be identical to the minimum value of h predicted from the Hubbard/Dukler model. However, visual studies show that under the condition of slug formation, the observed minimum film thickness may be different from that calculated from the Hubbard/Dukler model. An alternate speculation is based on the observation that h_s is equivalent to the neutral stability level, that is, the level below which no waves will form at the interface. Large waves are never seen when the liquid level reaches its lowest point. A model for this stability level was recently developed by Taitel & Dukler (1976) and is one which satisfies the condition

$$u_G \equiv \frac{W_G}{\rho_G A_G} = C\left[\frac{(\rho_L - \rho_G) g \cos \alpha A_G}{\rho_G A_L'}\right]^{1/2} \qquad [14]$$

where

$$C = 1 - h_s/D. \qquad [15]$$

Equation [14] with $C = 1$ is the condition for neutral stability for infinitesimal disturbances and is equivalent to the instability criterion given by [13]. However as explained by Taitel & Dukler (1976), for finite disturbances the use of [15] for C is more appropriate. For specified gas and liquid flow rates, pipe diameter and fluid properties all terms in [14] and [15] depend uniquely on h_s and this level, which is termed "the stability level", thus can be calculated directly. While the use of h_s is based on observation and not theory, it is consistent with the physical notion of the mechanism for formation and initial movement of a slug. When a large amplitude wave is present on the surface, it propagates rapidly downstream due to upstream gas velocity and tends to sweep the liquid in front of it. As a result, the liquid level continues to drop to satisfy conditions of continuity. However, once large waves can no longer exist on the film interface, the level is controlled solely by the forces on the planar interface and by the flow rates which exist. Then rebuilding of the film can begin. This stability level displays variations with gas and liquid rates which coincide with observation. It decreases with increasing gas rate out as expected and when $h_s > h_e$ no slugging can take place which is consistent with experiment.

It is now possible to summarize the boundary and initial conditions for this problem

$$\begin{aligned} t=0 \quad x>0 \quad h=h_s \quad u_L=u_e \\ t\geq 0 \quad x=0 \quad h=h_e \quad u_L=u_e. \end{aligned} \qquad [16]$$

Method of solution

Equations [3] and [4] are two simultaneous partial differential equations in $h(x,t)$ and $u_L(x,t)$. The R.H.S. of [3] is a known function of h and u_L and the equations are of the hyperbolic type subject to numerical solution by the method of characteristics. However, it is more convenient to use finite difference techniques with a simple (x,t) grid so that $h(x,t)$, $u_L(x,t)$ is given for any preselected time.

The specific finite difference scheme selected and the boundary conditions depend on whether the liquid film flow is tranquil (characteristics have positive and negative slope) or supercritical, where characteristics have only positive slope. The condition for supercritical flow in a two dimensional channel is $\sqrt{(gh)} < u_L$. For a circular tube the equivalent expression is $\sqrt{(gA_L/A_L')} < u_L$. Calculations demonstrate that for conditions of slug formation the flow is always supercritical. As a result the value of both the liquid elevation, h, and velocity, u_L, can be set at the inlet as given by the boundary conditions [16], (contrary to tranquil flow where only one h or u_L can be set at the inlet).

The simplest numerical scheme is one of a first order accuracy and is described by Stoker (1957). In this scheme one uses backward differentiation for calculating the derivatives with respect to x when both characteristic lines have a positive slope (which is the case for supercritical flow). For the time derivative forward difference is employed leading to a simple explicit numerical method. The time increment Δt is chosen so that each new time is less than the time given by the characteristic line in order to insure stability of the numerical scheme. Δx is chosen in successively smaller increments until convergence is achieved. A second order accuracy numerical scheme which may be used is based on the Lax–Wendroff (L–W) scheme as described for examples in Richtmyer & Morten (1967) or Mitchell (1969).

In this work the first order scheme was used. The method is simpler, it handles shocks more smoothly compared to the L–W scheme which produced oscillations before and after the shock and its accuracy was comparable to the L–W scheme for the same computation time.

Consider liquid and gas entering the pipe and the stratified flow that results in the entry region. A steady state solution of [3] and [4] exists and it shows that the elevation of the liquid in the pipe will gradually change to the equilibrium elevation h_e for which case the pressure drop in the gas is equal to that of the liquid. Figure 2 shows one specific solution to such a profile. However, as discussed above, when the interface is not stable waves will form and grow and will eventually block the gas passage. The shape of the film immediately after slug formation is approximated as a step change from the equilibrium elevation h_e to a film elevation h_s as shown in figure 2 for $t = 0$.

For $t > 0$ the liquid level rebuilds in a manner shown in figure 2. It is interesting to observe that there is a definite time at which the liquid level starts to overshoot the equilibrium level at a position which is farther downstream than the original step change at $x = 0$. The time it takes the liquid to reach the equilibrium level is considered to be the time for slug formation. Although waves may form on the rebuilding liquid, a stable slug is not formed unless the liquid level reaches h_e. After slug formation the liquid film drains again, a step shape profile as shown for $t = 0$ is assumed and the rebuilding cycle starts over again. Note that because the flow is supercritical the steady state levels located to the left of the step change ($x < 0$) are unaffected by the process taking place downstream ($x = 0$) and this is as observed experimentally. Thus the unsteady state problem need be solved only for $x \geq 0$. The slug in formed at a point where the steady state levels have already been reached and this suggests that slug frequency is independent of entrance geometry. This has been observed (Hubbard 1965).

For inviscid flow this case is similar to the classical "breaking dam problem" (Stoker 1957) superimposed on a uniform velocity. Inspection of such a solution shows that a step change caused by a breaking dam develops into a profile which contains a "simple wave" followed by a zone of constant level followed by a shock. The solution described here, although for circular geometry, would be expected to follow the same pattern. A solution obtained for the case of inviscid flow showed that this numerical technique tends to smear out the discontinuities in h and its gradient. Nevertheless, for the purpose of this model the simple finite difference scheme supplies sufficient accuracy in the $h(x, t)$ profile. The more complex second order scheme of Lax–Wendroff was attempted for this inviscid case but did not result in better accuracy.

DIMENSIONLESS REPRESENTATION

It is now possible to examine those dimensionless groups controlling the slug frequency which are compatible with this model. Consider the following normalizing variables: the tube diameter D for length, D^2 for area, the superficial velocities $u_L{}^s$ and $u_G{}^s$ for the liquid and gas velocities, respectively and $D/u_L{}^s$ for time. Introducing these variables into the differential equations and designating the dimensionless quantities by a tilde (˜) results in the following equations:

$$\frac{\partial \tilde{u}_L}{\partial \tilde{t}} + \tilde{u}_L \frac{\partial \tilde{u}_L}{\partial \tilde{x}} + \left[\frac{1}{F_L{}^2}\right]\frac{\partial \tilde{h}}{\partial \tilde{x}} + \left[\frac{X^2}{8Z^2}\right]\frac{\tilde{S}_L}{\tilde{A}_L}\left(\frac{4\tilde{u}_L\tilde{A}_L}{\tilde{S}_L}\right)^{-n_L}\tilde{u}_L{}^2$$
$$- \left[\frac{1}{8Z^2}\right]\left[\frac{\tilde{S}_i}{\tilde{A}_L} + \frac{\tilde{S}_i}{\tilde{A}_G} + \frac{\tilde{S}_G}{\tilde{A}_G}\right]\left[\frac{4\tilde{u}_G\tilde{A}_G}{\tilde{S}_G + \tilde{S}_i}\right]^{-n_G}\tilde{u}_G{}^2 - \left[\frac{Y}{2Z^2}\right] = 0, \qquad [17]$$

$$\frac{\partial \tilde{h}}{\partial \tilde{t}} + \tilde{u}_L \frac{\partial \tilde{h}}{\partial \tilde{x}} + \frac{\tilde{A}_L}{\mathrm{d}\tilde{A}_L/\mathrm{d}\tilde{h}}\frac{\partial \tilde{u}_L}{\partial \tilde{x}} = 0. \qquad [18]$$

Equations [17] and [18] are the momentum and continuity differential equations for $\tilde{h}(\tilde{x}, \tilde{t})$ and $\tilde{u}_L(\tilde{x}, \tilde{t})$ and their solution depends on the parameters F_L, Z, X, and Y.

The dimensionless equilibrium level $\tilde{h}_e$ which is used as a boundary condition can be obtained from [17] by setting the derivative terms to zero. In this case an algebraic equation results for $\tilde{h}_e$ whose value can be seen to depend only on the parameters X and Y (Taitel & Dukler 1976). The dimensionless stability level is obtained from [14] and [15] to yield

$$F_G = \frac{1 - \tilde{h}_s}{\tilde{u}_G} \frac{\tilde{A}_G}{[\mathrm{d}\tilde{A}_L/\mathrm{d}\tilde{h}]^{1/2}}. \qquad [19]$$

Thus the dimensionless frequency will depend on

$$\frac{\nu D}{u_L{}^s} = f(X, Z, F_L, F_G, Y), \qquad [20]$$

where:

$$X = \left[\frac{(4C_L/D)(u_L^s D/\nu_L)^{-n_L}\rho_L(u_L^s)^2/2}{(4C_G/D)(u_G^s D/\nu_G)^{-n_G}\rho_G(u_G^s)^2/2}\right]^{1/2} = \left[\frac{(\mathrm{d}P/\mathrm{d}x)_L^s}{(\mathrm{d}P/\mathrm{d}x)_G^s}\right]^{1/2}, \quad [21]$$

$$Z = \sqrt{\left(\frac{\rho_L(u_L^s)^2/2D}{|(\mathrm{d}P/\mathrm{d}x)_G^s|}\right)}, \quad [22]$$

$$F_L = \frac{u_L^s}{\sqrt{(gD\cos\alpha)}}, \quad [23]$$

$$F_G = \frac{u_G^s}{\sqrt{(Dg\cos\alpha)}}\sqrt{\left(\frac{\rho_G}{\rho_L-\rho_G}\right)}, \quad [24]$$

$$Y = \frac{g(\rho_L-\rho_G)\sin\alpha}{(\mathrm{d}P/\mathrm{d}x)_G^s}. \quad [25]$$

In this set of dimensionless groups X is the Lockhart–Martinelli Parameter and F_L is the liquid Froude Number. Z represents the ratio of inertial to gas phase pressure drop forces. The F_G number emerges from nondimensionalizing of [14] and measures the ratio of inertial forces of the gas to gravity forces on the liquid liquid. Y is zero for horizontal tubes and represents the relative forces acting on the liquid in the flow direction due to gravity and pressure drop. The superscript s is used to indicate a superficial condition, that is, the value which would be calculated if that one phase flowed alone in the pipe.

In a recent paper Grescovich & Shrier (1972) suggested that the slug frequency depends on only two dimensionless groups: the mixture Froude Number $F_r = (u_L^s + u_G^s)^2/Dg$ and the input liquid quality volume fraction $\lambda = u_L^s/(u_L^s + u_G^s)$. The authors do not include, for example, the data of Gregory & Scott (1969) for 1.9 cm pipes or those of Vermeulen & Ryan (1971) for 1.27 cm pipes. These data are underpredicted by a factor of two by their correlation. By comparison this analysis which is based on a physical model shows that the frequency for horizontal tubes depends on four dimensionless groups none of which are similar to those of Grescovich & Shrier (1972). Grescovich & Shrier's presentation is not consistent in the sense that their two dimensionless parameters are used to correlate a dimensional frequency rather than a dimensionless one.

In this work numerical solutions were generated in dimensional form to compare with the experimental data available. An example of the solution of the dimensionless form of the equation is also shown.

COMPARISON OF THEORY WITH DATA

Experimental data were reported by Hubbard & Dukler (1975) for air–water in a 3.81 cm diameter pipe, by Gregory & Scott (1969) for air–CO_2 in a 1.9 cm pipe and by Vermeulen & Ryan (1971) using air–water in a 1.27 cm diameter pipe. These data are presented in figures 3, 4 and 5 by a solid line which represents their best fit curve to the experimental data. The results calculated from this theory are shown by the dotted lines.

Comparison of this theory with the Hubbard/Dukler data appears in figure 3 where the agreement is seen to be good both in the trend of prediction and in the absolute value of the frequency calculated. The theory predicts increasing frequency with liquid rate for any single gas rate as observed. It also displays a minimum in the frequency curve with gas rate at constant liqud flow as shown by the data. It should be emphasized that the frequency data are in no way used in the theory to adjust or position the curves, the only empiricism being the use of the well accepted Blasius relationship for calculating the wall shear.

Agreement of theory with the data of Gregory and Scott as shown in figure 4 is not as satisfactory but still quite acceptable, seldom exceeding 25%. The theory again displays the correct trend of the data. It should be noted that measurement of frequency is difficult to

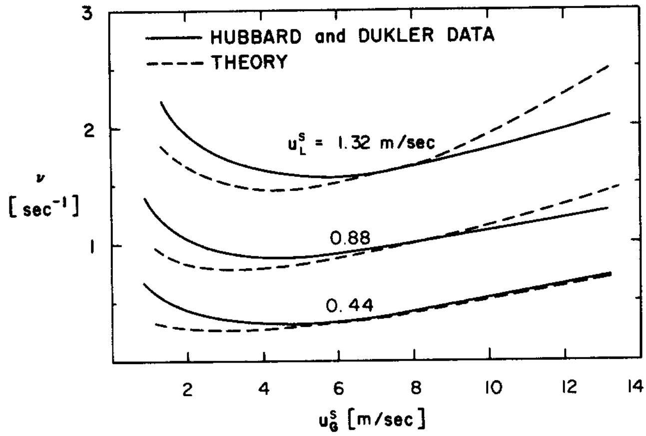

Figure 3. Comparison of theory with data, water–air, 3.81 cm pipe diameter.

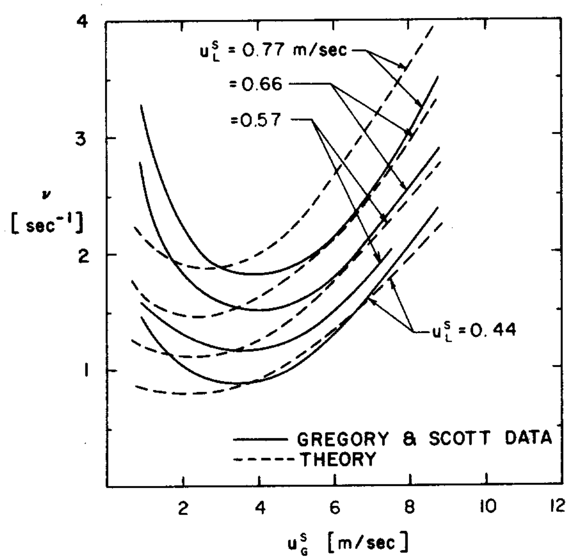

Figure 4. Comparison of theory with data, water–CO_2, 1.9 cm pipe diameter.

accomplish with accuracy. There is a factor of judgement involved, especially at higher liquid rates in deciding whether a highly aerated slug should be counted or not. In addition, when operating near the transition boundaries between slug and annular flows the difference between a slug and a roll wave is difficult to distinguish.

The agreement between the theory and data of Vermeulen is less satisfactory as shown in figure 5. Note that the theory predicts the increase of frequency with gas velocity at all gas velocities above the minimum but the data do not display this trend. This trend is in contradiction with the data of Hubbard and of Gregory since it is highly unlikely that the slugging behavior will be markedly different between 1.9 cm and 1.27 cm diameter pipe. It is suspected therefore that Vermeulen's data are questionable, especially for high gas rates.

This theory will not be able to predict frequency when slugs are created by entrance sections of unusual design which do not permit stratified flow to be established as the entry region. For example, Chu (1968) measured frequency in a 1.9 cm diameter pipe. He introduced his air through a tube located axially extending into the test section. At low gas rates this jet type entry appears to have suppressed slug formation and the characteristic minima differ markedly from Gregory's data.

Similarly, Kordyban (1963) reported slug frequencies for air–water flow at rates where slug

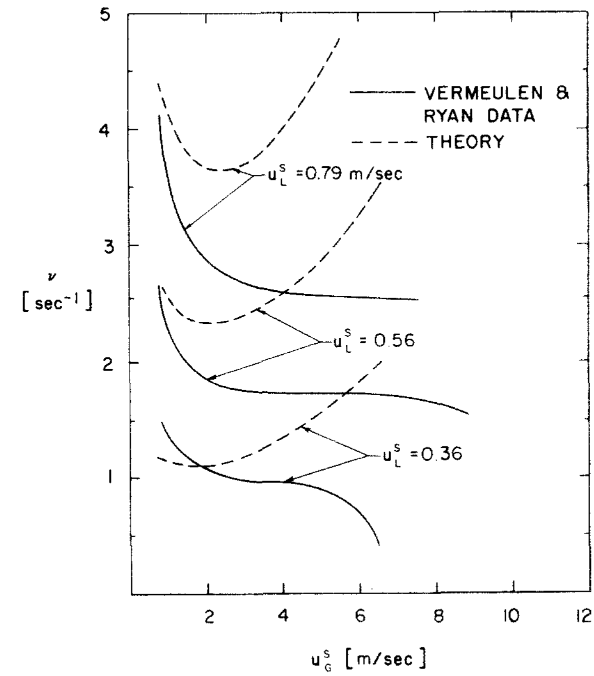

Figure 5. Comparison of theory with data, water–air, 1.27 cm pipe diameter.

flow would not be expected. However, he induced slugs by forcing the fluids through a U tube. The slugs thus formed persisted only over a small length of pipe where he was able to observe their presence. This slug frequency likewise cannot be predicted by the theory presented here.

GENERALIZED SOLUTION

It is advantageous to represent the results for the frequency in a dimensionless form, so that one can avoid making a computer solution for every case of interest. However since the dimensionless frequency depends on five parameters it is laborious to cover the whole range of possible operating conditions. The experiments discussed above include 3 pipe sizes for water–air or CO_2 systems. For these conditions the values of the parameters vary as follows:

F_L: 0.7–2.2;
Z: 6–120;
X: 1–20;
Y: 0–0;
F_G: 0.08–0.9;

whereas the result for the dimensionless frequency ranges from 0.05–0.2. The range of values of dimensionless frequency is narrow compared to the range of the dimensional frequencies. This is a direct result of the parameter selected to scale the frequency, $\tilde{\nu} = \nu D/u_L^s$. Since the frequency increases quite strongly with decreasing pipe diameter and increasing liquid flow rate, the dimensionless frequency is a weaker function of those variables than is its dimensional value.

Figure 6 shows a typical solution in dimensionless form for fixed values of the parameters $F_L = 2$, $Z/X = 5$ and $Y = 0$. The results are given in terms of Z/X which equals $(4f_L^s)^{-1/2}$ since this grouping is weakly dependent on operating conditions. The locus of the transition boundaries to stratified flow are shown at the left of each curve by a short vertical line. These were determined from the correlation of Taitel & Dukler (1976). For $Y = 0$ annular flow will

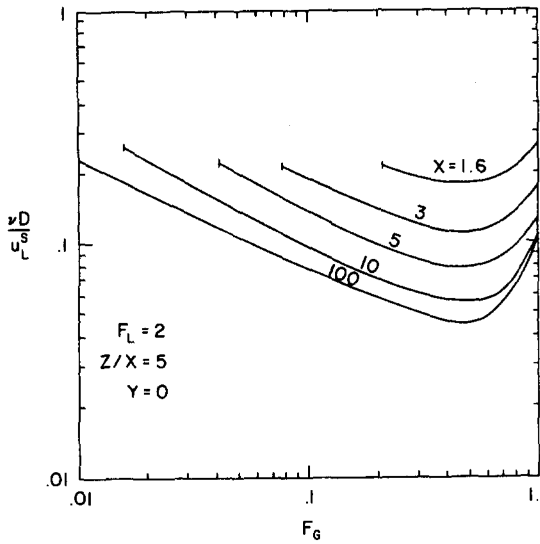

Figure 6. Dimensionless slug frequency.

exist below $X = 1.6$. Thus this dimensionless representation displays both the frequency of the slug and the conditions when slug flow can exist.

CONCLUSIONS

A fundamental model has been presented which is shown to predict slug frequency for entry sections in which natural slugging is permitted to take place. The agreement with experimental data is shown to be within probable limits of data uncertainty. Five dimensionless groups are shown to control the dimensionless frequency, $(\nu D/u_L^s)$.

Acknowledgement—The financial support of the American Institute of Chemical Engineers, Design Institute for Multiphase Processing is gratefully acknowledged.

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